SPATIAL VARIATION OF NONWOVENS SURFACE DENSITY

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1. INTRODUCTION

The products from nonwovens are nowadays applicable in the fields requiring relative high mass uniformity or uniformity of basic physico - mechanical properties. There exists a lot of methods for description of planar anisotropy and other structural characteristics of nonwovens [1,2]. Selected methods of continuous and discontinuous measurement of planar uniformity of nonwovens are described in the dissertation [4]. Basic on is direct measurement of local planar mass variation by weighting (gravimetric method). The extension of linear textiles uniformity based on the variation coefficient CV to the planar case is frequently used [2].

In this contribution the planar uniformity is described on the base of random field theory and spatial autocorrelation indices.

2. RANDOM FIELD OF PLANAR UNEVENNESS

The planar density z(x,y) describes sufficiently the planar uniformity or unevenness [2]. The quantity z(x,y) in thec point x,y is defined as limit of mass M(S) divided by the area S =4dxdy of elementary rectangle i.e. the cross sectional area of volume element having thickness t (thickness of nonwoven) and perpendicular dimensions x ± dx and y ± dy. Formally

$$z(x,y) = \lim_{S \to 0} \frac{M(S)}{S} = t * r(x,y) \qquad (1)$$

where ρ(x,y) is planar textile density in the point x,y. Quantity z(x,y) is random function of two variables called random field. This random field is fully described by the n variate probability density function

$$p_n(z_1, z_2, .. z_n) = P\{z_i \le z(x_i, y_i) \le z_i + dz_i, \quad i = 1..n\} \qquad (2)$$

Homogeneous random field has property of invariance according to the translation. *Variability* of random field is characterized by the correlation function

$$R(x_1, x_2, y_1, y_2) = \iint (z_1 - E(z_1)(z_2 - E(z_2)\, p_2(z_1, z_2)\, dz_1\, dz_2 \qquad (3)$$

The mean value $E(z_1)$ is defined as

$$E(z) = \int z\, p_1(z)\, dz \qquad (4)$$

For *homogeneous random field* is correlation function dependent on the distance between poins (x_1,y_1) and (x_2,y_2) only. For this case is valid

$$R\,(x_1, x_2, y_1, y_2) = R\,(x_2 - x_1, y_2 - y_1) \qquad (5)$$

For isotropic random field is correlation function invariant against rotation and mirroring . This function is then dependent on the length $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and therefore

$$R\,(x_1, x_2, y_1, y_2) = R\,(d) \qquad (6)$$

For computation of correlation function the experimentally determined values of planar densities z(i,j) of i,j th cell (i = 1...m, j = 1...n) of the rectangular net are used. The estimate of the correlation function is then defined by the relation [3]

$$R\,(K, L) = \frac{1}{(m-K)(n-L)-1} \sum_{i=1}^{m-K} \sum_{j=1}^{n-L} (z(i+K, j+L) - \bar{z})(z(i,j) - \bar{z}) \qquad (7)$$

The arithmetic mean of surface density has the form

$$\bar{z} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} z(i,j) \qquad (8)$$

It is simple to derive that correlation function R(0,0) = D(z(i,j)) is equal to the variance D(z(i,j)) of the surface density. For the Gauss random field is correlation function dependent on the differences $d_x=x_2-x_1$ a $d_y=y_2-y_1$ only [2]

$$R(d_x, d_y) = \exp(-a|d_x| - b|d_y|) \qquad (9)$$

For characterization of the random field non homogeneity the anisotropy coefficient An defined by relation

$$An = \frac{Km}{Lm} \qquad (9)$$

is used. The Km and Lm correlation are intervals i.e. minimal values K and L for which is valid

$$R(Km,0) \le 0.05 * R(0,0) \quad \text{resp.} \quad R(0,Lm) \le 0.05 * R(0,0) \qquad (10)$$

The parameters Km and Lm are connected with parameters of the Gauss random field by relations

$$a = \frac{3}{Km} \quad \text{and.}\; b = \frac{3}{Lm} \qquad (11)$$

Based on the estimate of correlation function is therefore straightforward to describe anisotropy characteristics An. For estimation of the correlation function and computation of anisotropy characteristics the program in MATLAB 5.3 language has been created.

2. SPATIAL AUTOCORELATION

The spatial randomness in the plane can be expressed by the spatial autocorrelation indices. For definition of spatial autocorrelation some measure of contiguity is required. Simple contiguity measures are defined as neighborhood relations. The kings case considering neighborhood of eight cells was used in this work. The connectivity (spatial weight) matrix W contains

elements Wij = 1, if i-th and j-th cell are neighborhood or Wij = 0 if i-th and j-th cell are far each other.
Let the value Zk = z(i,j) for k = i + m*(j-1) are surface densities z(i,j) arranged columnwise. The Geary autocorrelation index is defined by relation [3]

$$c = \frac{N-1}{2*\sum_i \sum_j W_{ij}} * \frac{\sum_i \sum_j W_{ij} * (Z_i - Z_j)^2}{\sum_i (Z_i - Zm)^2} \quad (12)$$

where Zm is arithmetic mean of all cells surface densities. The statistics c is defined in the range from 0 to 2. Negative spatial autocorrelation is for c>1 and positive spatial autocorrelation is for c<1. Mean value (spatial randomness) is equal to E(c) = 1. Variance D(c) based on the approximate normality is defined as

$$D(c) = \frac{(N-1)*(2*S_1 + S_2) - 4*S_0^2}{S_0^2 * 2*(N+1)} \qquad (13)$$

Individual symbols in eqn. (13) are defined as

$$S_0 = \sum_i \sum_j W_{ij} \quad S_1 = \frac{1}{2}\sum_i \sum_j (W_{ij} + W_{ji})^2 \quad S_2 = \sum_i (W_{i*} + W_{*i})^2 .$$

Symbol Wi* denotes i-th row and W*i denotes i-th column of matrix W. Random variable

$$Z(c) = \frac{c-1}{\sqrt{D(c)}}$$

has approximately standardized normal distribution. If absolute value abs(Z(c))≥2 the significant autocorrelation occurs

3. EXPERIMENTAL PART

The chemically bonded (by the acrylate binder) nonwoven from viscose fibers (VS) was prepared. Starting lap of planar weight 60 g m^{-2} was created on the pneumatic web former. The lap consists of two types of viscose fibers mixed in the weight ratio 67/33 (VS 3,1 dtex/60 mm and 1,6 dtex/40mm). Binding acrylate (relative amount 20 %) was applied by padding

The rectangular samples of dimensions 100 x 100 mm (area Aj = 100 mm^2 and weight 6 mg) were cut for further analysis [3].These samples were divided to the rectangular net having dimensions of individual cells 10 x 10 mm. Relative error of cell dimensions was in the interval from 0.88% to 1,22%. The weight m_{ij} of i,j th cell has been computed as mean from five parallel weightings. Maximal relative error of weighing was 1,606%.

4. RESULTS AND DISCUSSION

From the weights m_{ij} and cell area $S_j = 100$ mm^2 the surface densities $z_{ij} = m_{ij} / S_j$ v [g m^{-2}] have been computed. Basic statistical characteristics of resulted random field of surface density are given in the table 1

Table 1. Basic characteristics of surface density

Number of values	100	dimension
Mean	58.92	[g m^{-2}]
Standard deviation	5,12	[g m^{-2}]
Variation coefficient	8.68	[%]

The intervals of correlation are Km=3 a Lm=2 and therefore An=1.5.The bivariate autocorrelation unction R(K,L) for K=0,1.....7 and L=0,1...7 is given on the fig.2.

It is clear that the random field of surface density is slightly anisotropic and has local nonregularities. The index of anisotropy is not so far from one. The Geary index c = 0.768, D(c)= 0.0037and the standard normal one is Z= -3.79 The value of c is below 1 and therefore the random variation has to be rejected. The positive autocorrelation shows that the values of surface densities z(i,j) in adjacent cells are directly associated. The spatial randomness has to be therefore omitted

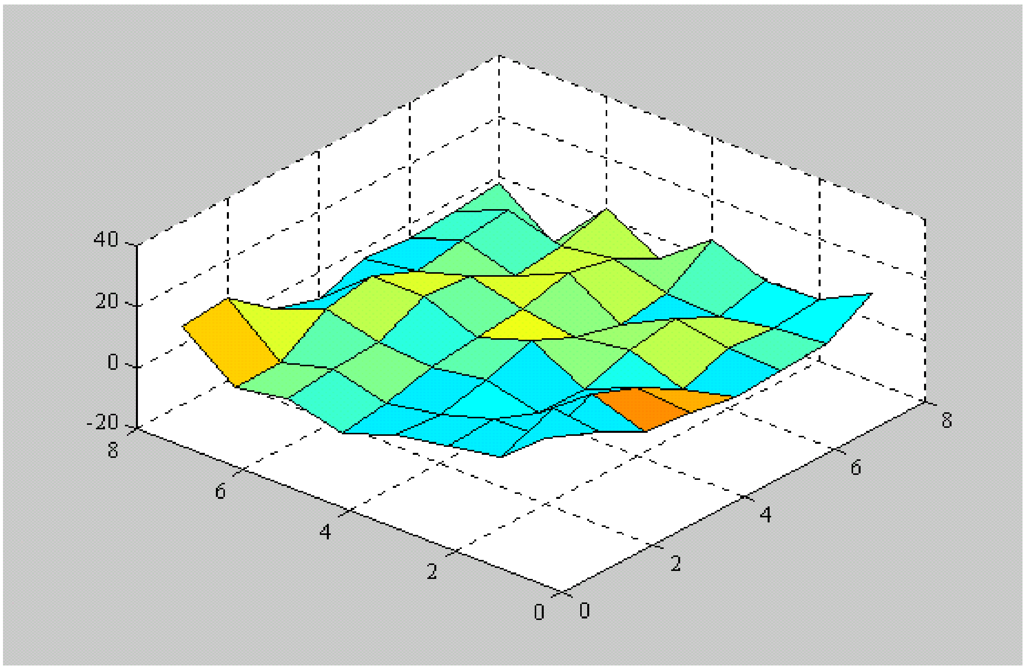

Fig. 2 Bivariate autocorrelation function R(K,L)

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